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DIFFRACTION OF A SHOCK WAVE ON A WEDGE MOVING AT SUPERSONIC SPEED

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We investigate the differentiation of a shock wave of an arbitrary intensity on the upper surface of a wedge moving at supersonic speed under the assumption that the difference between the intensities of the shock wave and the attached shock as well as the difference between the wedge angle α and the angle of incidence of the shock wave δ are both small (Fig. 1).

The case of a flow when a plane shock wave impinges on a wedge moving at supersonic speed and diffraction is absent, was dealt with in [1]. In the present paper we obtain conditions under which a constant parameter flow is realized in the region AFK bounded by the impinging shock wave, the attached shock and the wedge wall.

Diffraction of a shock wave of arbitrary intensity on a slender wedge moving at supersonic speed was dealt with in [2]. Paper [3] was concerned with the diffraction of a weak wave on a slender wedge moving at hypersonic speed. In addition, diffraction of a weak wave on an arbitrary wedge moving at supersonic speed was the theme of a Candidate's

Dissertation of S. M. Ter-Minasiants entitled "Diffraction of a plane wave on a wedge moving at supersonic speed", MGU, 1967.

1. Statement of the problem. Superposition of perturbations on a constant parameter flow yields a diffraction pattern bounded by the shock wave AB, the shock CD, the wedge wall AE and the arcs BC and DE of the Mach circle whose center O moves along the wedge wall at a velocity equal to the velocity of flow $U_0 - U$ behind the shock wave AF, where U_0 is the velocity of the shock wave and U is the velocity of the shock wave relative to the flow behind it. We shall formulate the problem in the $Ox'y'$ coordinate system with its origin O at the moving center of perturbations which coincides with the point of intersection of the bisector of the angle AFK with the wedge wall. In the system thus defined, the unperturbed gas within the region AFK will be at rest and the problem will be self-similar with respect to time t. Let us linearize the equations of a two-dimensional, unsteady motion of gas and introduce the following dimensionless variables:

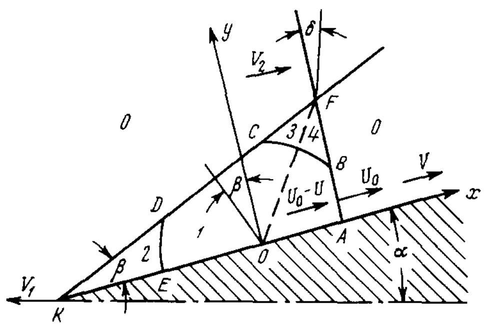

Fig. 1

$$u = \frac{u'}{a_1}, \quad v = \frac{v'}{a_1}, \quad p = \frac{p'}{a_1^2\rho_1}, \quad x = \frac{x'}{a_1 t}, \quad y = \frac{y'}{a_1 t}$$

where u', v' and p' represent the perturbed velocity components and the pressure perturbation, while a_1 and ρ_1 are the velocity of sound and the unperturbed density in the region 1.

Let us write the equation of the perturbed shock wave front AF in the form

$$x = k + \psi(y)$$

For $x = k$, the relations at the shock wave AB become

$$u = \frac{2}{\gamma+1}\frac{(M_0-M)^2+1}{(M_0-M)^2}(\psi(y) - y\psi'(y)) + D_1 \tag{1.1}$$

$$v = (k - m(M_0 - M))\psi'(y) - v_0, \qquad p = \frac{4k}{\gamma+1}(\psi(y) - y\psi'(y)) + E_1$$

$$\left(M_0 = \frac{U_0}{a_0}, \quad M = \frac{V}{a_0}, \quad k = \frac{U}{a_1}, \quad m = \frac{a_0}{a_1}\right)$$

where a_0 and V are the velocity of sound and the stream velocity in the region O, and D_1 and E_1 are known constants dependent on the perturbed parameters u_0, v_0, ρ_0 and p_0 in the region O ahead of the shock wave.

Equations (1.1) can be written as

$$u = A_1 p + F_1, \quad y\frac{\partial v}{\partial y} = B_1 \frac{\partial p}{\partial y} \quad \text{for} \quad x = k \tag{1.2}$$

$$A_1 = \frac{(M_0-M)^2+1}{2k(M_0-M)^2}, \quad B_1 = \frac{\gamma+1}{2}\frac{(M_0-M)^2-1}{(\gamma-1)(M_0-M)^2+2}$$

where F_1 is a known constant and γ is the ratio of specific heats.

Analogous relations can be set up at the shock KF for $y\cos\beta - x\sin\beta = k$.

The flow in the region 2 is known and corresponds to a flow past a wedge at a velocity $\mathbf{V}_2 - \mathbf{V}_1$, where $\mathbf{V}_1$ is the velocity of the wedge and $\mathbf{V}_2$ is the velocity of flow behind the impinging shock wave.

Constant parameter flows in the regions 3 and 4 are completely defined by writing the relations (1.1) on the slightly deviating rectilinear segments:

of the shock wave FB

$$x = k - (y - y_F) \operatorname{tg} \varepsilon_1$$

of the shock FC

$$y \cos\beta - x \sin\beta = k + (y - y_F) \operatorname{ctg} (\beta - \varepsilon_2)$$

together with the following condition of the weak tangential discontinuity FO

for $y = x \operatorname{tg} {}^1/_2\, \theta_0$ $\quad u_3 y_F - v_3 x_F = u_4 y_F - v_4 x$

$$(x_F = k,\ y_F = k \operatorname{tg} {}^1/_2 \theta_0,\ \theta_0 = {}^1/_2\, \pi + \beta)$$

on which the pressure and its derivatives are constant [2].

The above relations yield seven conditions defininf seven unknowns u_3, u_4, v_3, v_4, ε_1, ε_2 and $p_3 = p_4$, where ε_1 and ε_2 denote small angles of deviation of the impinging shock wave and of the attached shock.

2. Formulation of the problem for the function p . Linearization and the Chaplygin transformation reduce the problem to the Laplace's equation for the pressure perturbation. The region corresponding to the diffraction region will map into an orthogonal curvilinear pentagon $ABCDE$ on the plane $z = r \exp i\theta = \mu + i\nu$ bounded by four circular arcs and a straigh line (Fig. 2).

Boundary conditions for the normal and tangential partial pressure derivatives will be [4]

$$a \frac{\partial p}{\partial n} + b \frac{\partial p}{\partial s} = 0$$

Here

$$\begin{array}{lll} a = \vartheta(\theta, 0), & b = 1 & \text{on } AB \\ a = 0, & b = 1 & \text{on } BC \text{ and } DE \\ a = \vartheta(\theta, \theta_0), & b = 1 & \text{on } CD \\ a = 1, & b = 0 & \text{on } AE \end{array}$$

and

$$\vartheta(\theta, 0) = \frac{\sqrt{1 - k^2 \sec^2\theta}}{kA_1 \operatorname{tg}\theta - B_1 \operatorname{ctg}\theta}, \qquad \vartheta(\theta, \theta_0) = \frac{\sqrt{1 - k^2 \sec^2(\theta - \theta_0)}}{kA_1 \operatorname{tg}(\theta - \theta_0) - B_1 \operatorname{ctg}(\theta - \theta_0)}$$

The respective equations of the circular arcs AB and CD have the form

$$k(1 + r^2) = 2r\cos\theta, \qquad k(1 + r^2) = 2r\cos(\theta - \theta_0)$$

The coefficient a accompanying $\partial p / \partial n$ becomes infinite at the points $N \in AB$, L and $Q \in CD$, for $\quad \theta_N = \operatorname{arc\,tg} \sqrt{B_1 / kA_1}, \quad \theta_{L,Q} = \theta_0 \mp \operatorname{arc\,tg} \sqrt{B_1 / kA_1}$

Integrating the second condition of (1.2) along the shock wave AB and considering that $y = k \operatorname{tg}\theta$ for $x = k$, we obtain

$$k^{-1} B_1 \int_{AB} \operatorname{ctg}\theta\, dp = v_B - v_A \tag{2.1}$$

while the conditions which must hold along the shock CD have the form

$$k^{-1} B_1 \int_{CD} \operatorname{ctg}(\theta - \theta_0)\, dp = (u_C - u_D)\cos\beta + (v_C - v_D)\sin\beta$$

$$\int_{CD} dp = p_2 - p_3 \tag{2.2}$$

Solution of the resulting boundary value problem is obtained by mapping the curvilinear pentagon $ABCDE$ into the upper semiplane.

3. Construction of the function mapping the curvilinear pentagon into the upper semiplane. Applying the bilinear transformation

$$\zeta = \frac{1-(k+ik_1)z}{z-k-ik_1}, \qquad k_1 = \sqrt{1-k^2}$$

we map the pentagon $ABCDE$ into the second quadrant, from which a quarter of the unit circle with its center at the origin, and a half-circle of radius d with its center on the real axis at the distance of $-c$ from the coordinate origin, are deleted (Fig. 3)

$$d = \frac{k_1}{k+k\sin\beta-k_1\cos\beta}, \qquad c = \frac{k^2+\sin\beta}{k+k\sin\beta-k_1\cos\beta}$$

The region obtained is then mapped into the upper semiplane by reflecting it into the first quadrant. The function which maps the region into the upper semiplane will be automorphic and

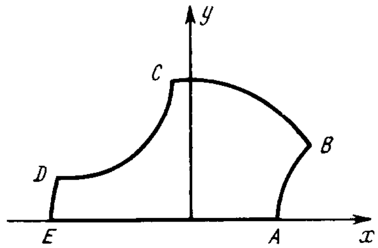

Fig. 2

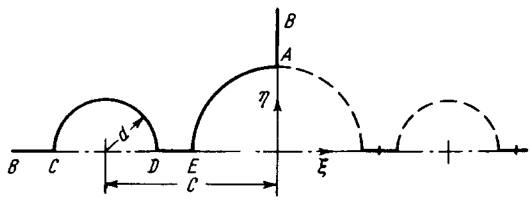

Fig. 3

given by the following analytic expression [5 and 6]:

$$\omega = \frac{\zeta}{2} + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\alpha_n z+\beta_n}{\gamma_n z+\delta_n} - \frac{\alpha_n}{\beta_n}\right) \tag{3.1}$$

Consequently ω is an automorphic function with an associated bilinear substitution group. The substitutions are obtained from all possible products of the basic substitutions

$$\begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}, \qquad \begin{Vmatrix} c & d^2-c^2 \\ 1 & -c \end{Vmatrix}, \qquad \begin{Vmatrix} -c & d^2-c^2 \\ 1 & c \end{Vmatrix}$$

Another general expression for the mapping function (3.1) can be represented in the form

$$\omega = \frac{1}{2}\left(\zeta+\frac{1}{\zeta}\right) + \frac{d^2\zeta}{\zeta^2-c^2} + \sum_{n=0}^{\infty}\frac{\mu_n\zeta}{\zeta^2-\xi_n^2} \tag{3.2}$$

$$\xi_n = \frac{\delta_n}{\gamma_n}, \qquad \mu_n = \frac{\beta_n\gamma_n-\alpha_n\delta_n}{\gamma_n^2} \qquad (n=0,1,2,\ldots)$$

Let us find the coefficients μ_n and ξ_n by computing a number of possible products of the basic substitutions

$$\xi_0 = \frac{1}{c}, \qquad \xi_{2n} = \frac{1}{\xi_{2n-1}}, \qquad \mu_0 = -\frac{d^2}{c^2}, \qquad \mu_{2n} = -\frac{\mu_{2n-1}}{\xi_{2n-1}^2} \quad (n=1,2,\ldots)$$

$$\xi_1 = c-\frac{d^2}{c}, \qquad \xi_3 = c-\frac{d^2}{2c}, \qquad \xi_{5,7} = c-\frac{d^2}{c\pm\xi_0}$$

$$\xi_9 = c-\frac{d^2}{c+\xi_1}, \qquad \xi_{11,13} = c-\frac{d^2}{c\pm\xi_2}, \qquad \xi_{15} = c-\frac{d^2}{c+\xi_3}$$

$$\xi_{17,19} = c - \frac{d^2}{c \pm \xi_4}, \qquad \xi_{21} = c - \frac{d^2}{c + \xi_5}, \qquad \xi_{23,25} = c - \frac{d^2}{c \pm \xi_6}$$

$$\xi_{27} = c - \frac{d^2}{c + \xi_7}, \qquad \xi_{29,31} = c - \frac{d^2}{c \pm \xi_8}, \qquad \xi_{33} = c - \frac{d^2}{c + \xi_9}$$

$$\mu_1 = -\frac{d^2}{c^2}, \qquad \mu_3 = -\frac{d^4}{4c^2}, \qquad \frac{\mu_{5,7}}{\mu_0} = \frac{-d^2}{(c + \xi_1)^2}$$

$$\frac{\mu_9}{\mu_1} = \frac{-d^2}{(c + \xi_1)^2}, \qquad \frac{\mu_{11,13}}{\mu_2} = \frac{-d^2}{(c \pm \xi_2)^2}, \qquad \frac{\mu_{15}}{\mu_3} = \frac{-d^2}{(c + \xi_3)^2}$$

$$\frac{\mu_{17,19}}{\mu_4} = \frac{-d^2}{(c \pm \xi_4)^2}, \qquad \frac{\mu_{21}}{\mu_5} = \frac{-d^2}{(c + \xi_5)^2}, \qquad \frac{\mu_{23,25}}{\mu_6} = \frac{-d^2}{(c \pm \xi_6)^2}$$

$$\frac{\mu_{27}}{\mu_7} = \frac{-d^2}{(c + \xi_7)^2}, \qquad \frac{\mu_{29,31}}{\mu_8} = \frac{-d^2}{(c \pm \xi_8)^2}, \qquad \frac{\mu_{33}}{\mu_9} = \frac{-d^2}{(c + \xi_9)^2}$$

etc. The law of formation of the coefficients is obvious. Rate of convergence of the series is inversely proportional to the value of the parameter d / c which appears in (3.2).

The problem solved here is that of the flow past three cylinders whose radii are d, 1 and d. Function $\omega(\xi)$ represents the complex velocity potential, μ_n denotes the doublet strength and ξ_n is the coordinate enclosed by that doublet. Doublet image intensity decreases rapidly with decreasing d / c. Replacing in the expressions for μ_n the squares with the cubes, we obtain the potential flow past three spheres.

A particular case of this problem, namely, a flow past two spheres, has already been solved by Stokes who used the method of consecutive approximations and placed the doublets of the given strength μ_n at the points of inversion ξ_n relative to the two spheres [7].

Final expression for the function mapping the initial curvilinear pentagon into the upper semiplane, has the form

$$w = f(z) = -\omega^2 \left(\frac{1 - (k + ik_1) z}{z - k - ik_1} \right)$$

4. Formulation and solution of the Hilbert problem. Let us introduce the function

$$P(w) = \frac{\partial p}{\partial \sigma} + i \frac{\partial p}{\partial \tau}$$

regular in the upper semiplane $w = \tau + i\sigma$ and satisfying the following condition on the real axis:

$$a(\tau) \frac{\partial p}{\partial \sigma} + b(\tau) \frac{\partial p}{\partial \tau} = 0$$

Here

$$\begin{array}{lll} a = 0, & b = 1, & -\infty < \tau < -(c + d)^2 \\ a = \vartheta(\theta, \theta_0), & b = 1, & -(c + d)^2 < \tau < -(c - d)^2 \\ a = 0, & b = 1, & -(c - d)^2 < \tau < -1 \\ a = 1, & b = 0, & -1 < \tau < 0 \\ a = \vartheta(\theta, 0), & b = 1, & 0 < \tau < \infty \end{array}$$

and

$$\operatorname{tg} \theta = \frac{\operatorname{Im} f^{-1}(\tau)}{\operatorname{Re} f^{-1}(\tau)} = \lambda(\tau), \qquad z = f^{-1}(w)$$

Coefficients a and b have first order discontinuities at the points $\tau = -1$ and $\tau = 0$. In addition, the coefficient a has second order discontinuities at the points $\tau_1 \in (0, \infty)$, τ_2 and $\tau_3 \in (-(c + d)^2, -(c - d)^2)$; the points τ_1, τ_2 and τ_3 correspond to the points

N, L and Q in the z-plane.

Substitution

$$P(w) = \frac{1}{\sqrt{w(w+1)}} P_1(w)$$

where $\sqrt{w(w+1)}$ is any branch regular in the plane and with a cut on the real axis, removes the first order discontinuities at $\tau = -1$ and $\tau = 0$ (see e.g. [4]). Hilbert problem is solved by reducing it to the Riemann's problem and the points τ_1, τ_2 and τ_3 become singular points [8 and 9]. Consequently the order of the problem is $\varkappa = 3$ and the solution of the Hilbert problem with a second order zero at infinity, has the form

$$P(w) = \frac{c_0 + c_1 w + c_2 w^2}{(w+i)^3 \sqrt{w(w+1)}} \exp \frac{1}{2\pi i} \int_{-\infty}^{\infty} \ln\left[\left(\frac{s+i}{s-i}\right)^{\varkappa} G(s)\right] \frac{ds}{s-w}$$

Here

$$\begin{aligned}
&G(\tau) = 1, && -\infty < \tau < -(c+d)^2 \\
&G(\tau) = \Theta(\theta, \theta_0), && -(c+d)^2 < \tau < -(c-d)^2 \\
&G(\tau) = 1, && -(c-d)^2 < \tau < 0 \\
&G(\tau) = \Theta(\theta, 0), && 0 < \tau < \infty
\end{aligned}$$

$$\Theta(\theta, 0) = \frac{1 - i\vartheta(\theta, 0)}{1 + i\vartheta(\theta, 0)}, \qquad \Theta(\theta, \theta_0) = \frac{1 - i\vartheta(\theta, \theta_0)}{1 + i\vartheta(\theta, \theta_0)}, \qquad \operatorname{tg}\theta = \lambda(\tau)$$

$$\ln\left(\frac{\tau+i}{\tau-i}\right)^{\varkappa} G(\tau) = \varkappa \ln\frac{\tau+i}{\tau-i} + \ln G(\tau)$$

where $\ln(\tau+i)(\tau-i)^{-1}$ implies a branch which varies continuously along the real axis (including the point at infinity) with exception of a certain point $\tau_0 \in (-\infty, \infty)$ different from any of the points of discontinuity of the coefficients a and b, and $\ln G(\tau)$ is defined by

$$\arg\frac{G(\tau_n - 0)}{G(\tau_n + 0)} = 0 \qquad (n = 1, 2, 3)$$

The real constants c_0, c_1 and c_2 are found from the conditions (2.1) and (2.2). In the z-plane the solution has the form

$$\frac{\partial p}{\partial \nu} + i\frac{\partial p}{\partial \mu} = \left(\frac{\partial p}{\partial \sigma} + i\frac{\partial p}{\partial \tau}\right) f'(z) = \frac{c_0 + c_1 f(z) + c_2 f^2(z)}{(f(z)+i)^3 \sqrt{f(z)(f(z)+1)}} \times$$

$$\times f'(z)\left(\exp\frac{3}{2\pi i}\int_{\Gamma} \ln\frac{f(s)+i}{f(s)-i}\,\frac{f'(s)\,ds}{f(s)-f(z)} + \right.$$

$$\left. + \exp\frac{1}{2\pi i}\int_{AB} \ln\Theta(\theta, 0)\frac{f'(s)\,ds}{f(s)-f(z)} + \exp\frac{1}{2\pi i}\int_{CD} \ln\Theta(\theta, \theta_0)\frac{f'(s)\,ds}{f(s)-f(z)}\right)$$

Γ — contour $ABCDE$

on AB $(0 < \theta < \theta_1,\ \theta_1 = \arccos k)$

$$r \exp i\theta = k^{-1}(\cos\theta - \sqrt{\cos^2\theta - k^2}) \exp i\theta$$

on CD $(\theta_0 - \theta_1 < \theta < \theta_0 + \theta_1)$

$$r \exp i\theta = k^{-1}(\cos(\theta - \theta_0) - \sqrt{\cos^2(\theta - \theta_0) - k^2}) \exp i\theta$$

and pressure is given by

$$p = \operatorname{Im}\int_{-1}^{z}\left(\frac{\partial p}{\partial \nu} + i\frac{\partial p}{\partial \mu}\right) dz + p_2 = \operatorname{Im}\int_{-1}^{w}\left(\frac{\partial p}{\partial \sigma} + i\frac{\partial p}{\partial \tau}\right) dw + p_2$$

With the pressure determined, the remaining unknown functions can also be found in their closed form. For example, the form of the diffraction shock wave AB can be computed from (1.1) according to the formula

$$\psi(y) = \frac{(\gamma+1)(y-k_1)}{4kk_1}E_1 + \frac{\psi(k_1)}{k_1}y - \frac{\gamma+1}{4k}y\int_{k_1}^{y} s^{-2}p(s)\,ds$$

where $\psi(k_1)$ is known from the solution of the problem in region 4.

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ON THE THEORY OF ELECTROMAGNETIC WAVE DIFFRACTION IN ACTIVE MEDIA

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The most effective methods of generation and intensification of electromagnetic waves are based on the interaction of beams of charged particles with attenuating media, in which use is made of the fundamental effect of a charged particle stream on the medium properties, with the latter changing from a passive state absorbing radiation to an active one intensifying the electromagnetic field. In particular, the essential difference between a passive and an active medium is confirmed by the fact that theorems related to fluctuating dissipation applicable to absorbing media do not hold in the case of active ones. Hence, it is to be expected that the electromagnetic wave diffraction in active media will also take place in a different manner.

Growth of the magnetic wave amplitude in an active medium is in fact limited by either nonlinear effects, or by the finite length of the system active section. In the latter